Active Control of Sound Transmission Through Elastic Plates Using Piezoelectric Actuators

E. K. Dimitriadis* and C. R. Fuller†
Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061

A theoretical investigation is undertaken concerning the potential of actively controlling sound transmission/radiation from a vibrating plate using piezoelectric elements bonded to the plate surface as actuators. The system analyzed consists of a plane acoustic wave incident on a baffled, clamped, thin circular plate. Sound transmitted through the plate is considered to be the noise field. An approximate theory is used to evaluate the degree of attenuation achieved when sinusoidally activated piezoelectric elements bonded to the panel are used to minimize the total radiated acoustic power. The results show that global attenuation of radiated sound can be achieved on or off plate resonance frequencies. Both the size and the location of the actuator are demonstrated to affect the degree of control achieved and spillover into residual modes. These results, combined with size, compactness, and cost aspects, reveal that piezoelectric elements have great potential in controlling structurally radiated sound.

Introduction

SOUND radiation from structural vibrations is an important noise problem. Representative examples are sound radiation from marine vessels and sound transmission through aircraft fuselage panels.[1] This paper is concerned with investigations on the control of structurally radiated sound by applying active forces directly to the vibrating structure.

Early theoretical and experimental work in active noise control is reviewed by Ffowcs-Williams[2] and Warnaka[3] for one-dimensional fields. Attempts of noise control of three-dimensional radiation fields are summarized by Magiante.[4] In all of these works the control transducers were point acoustic sources, which are essentially "spectrally white." The results are thus characterized by the need to use many acoustic control sources strategically positioned to reduce the ubiquitous problem of "control spillover."

For structurally radiated or transmitted noise, however, there is an attractive alternative to the use of active acoustic transducers. Since the noise field in these problems is directly coupled to the structural motion, it would be best if the control were applied directly to the radiating vibration field by force inputs. Fuller,[5,6] through analytical investigations, was the first to demonstrate that, by applying active point forces to a radiating panel, one can minimize the sound power radiated from the panel with only two actuators over a wide frequency range. The work reveals that it is only necessary to control structural motions (modes) that are well coupled to the radiated field. Fuller's results have recently been validated by experiments.[7]

However, there are still a number of disadvantages with point force actuators; mainly, they are obtrusive, cumbersome, and require support structures. Another potential problem is spillover into higher panel (nonradiating) modes of vibration. Recent work on novel concepts for both distributed sensing and driving transducers has opened new possibilities. Much of that work has concentrated on developing distributed actuators made of piezoelectric material to control beam vibrations. The most rigorous study of the stress-strain-voltage behavior of piezoelements bonded to or imbedded in beams was conducted by Crawley and de Luis.[8] They showed that the loads induced by piezoelements bonded to beams are effectively moments concentrated at the two ends of the actuators, when the bonding layer is very thin compared to the piezoelectric element and beam thickness. Another important result in that study was that these moments are independent of the actuator length.

Recently, a number of investigations have also experimentally demonstrated the potential of such actuators to simultaneously attenuate a number of beam vibration modes. Bailey and Hubbard[9] used a polymeric piezoelectric strip bonded along the whole length of a cantilever beam. Fanson and Chen[10] and Baz and Poh[11] used more efficient ceramic actuators to demonstrate simultaneous control of up to six modes with a single actuator.

Piezoelectric actuators offer a number of advantages that should be mentioned at this point. They are easily attached (bonded) to beams or plates, and they are very lightweight, cheap, and take much less space than sound sources or point actuators. In addition, they do not require a back-reaction support structure, which is often very difficult to achieve in a practical application.

It is apparent that the use of piezoelectric actuators can be extended to the control of two-dimensional vibrating systems such as panels and their associated radiation field. The present authors have recently theoretically demonstrated that two-dimensional rectangular piezoelectric elements can be used to selectively excite vibration modes in a rectangular plate.[12] They developed an extension of the one-dimensional work by Crawley and de Luis.[8] In those analyses the effective load induced by piezoelectric patches can be seen as line moments distributed along the boundaries of the actuators. Recently, Fuller et al.[13] have also experimentally demonstrated the feasibility of actively controlling vibration and sound radiation from panels by a piezoceramic patch that is bonded directly to the plate surface.

The present work centers on analytically investigating the sound control capabilities of piezoelectric actuators. The problem is that of controlling the sound radiated/transmitted through a baffled, thin, clamped, circular plate. A plane wave incident on one side of the plate is partially transmitted through panel vibrations; this transmitted wave is the primary field. A pie-shaped piezoelectric actuator is bonded to the plate surface and voltage activated with a sinusoidal signal of the same frequency as the noise. The goal of the analysis is to

Presented as Paper 89-1062 at the AIAA 12th Aeroacoustics Conference, San Antonio, TX, April 10–12, 1989. Received May 11, 1989; revision received March 12, 1990; accepted for publication July 24, 1990.

*Assistant Professor, Mechanical Engineering Department.
†Professor, Mechanical Engineering Department. Member AIAA.

calculate the needed complex amplitude of the voltage to the actuator that will modify the plate vibration so that the transmitted sound field is globally minimized. This is achieved by minimizing the total transmitted sound power from the panel. Through these analytical simulations it was possible to demonstrate that, indeed, such actuators can very effectively control the radiated sound at various frequencies on or off resonance. Results are presented for both the primary and minimized radiated sound fields as well as plate vibration distributions. It will be shown that both the size and the position of these actuators play an important role in the control.

Attention should be drawn to the fact that the following results are approximate and are strictly valid for small actuators where their shape can be seen as closely rectangular. The authors, however, believe that this analysis provides significant insights in the control action of these novel actuators.

Theory

Piezoelectric-Induced Vibrations

The loads induced by a piezoelectric element to an underlying thin plate to which it is bonded have been previously evaluated in rectangular coordinates.[12] It was shown in that study that the plate reacted with a uniform bending moment distribution in both directions and across the extent of the actuator. The effective external loads then turned out to be constant line moments along the actuator boundaries.

Here let us consider a thin, baffled, clamped circular plate of thickness $2h$ and radius a, with two identical pie-shaped piezoelectric elements of thickness t, bonded symmetrically on the two opposite surfaces of the plate as shown in Fig. 1. The two elements will have to be excited by opposite voltages if bending is to be induced. Clearly, the piezoelectric elements will generate distributed moment reactions in the plate. To the knowledge of the authors, this moment distribution has never been presented in the literature. A rigorous analysis of the problem in polar coordinates is under development and will be presented in the near future. However, in the present investigation the pie-shaped piezoelectric elements will be approximated as rectangular elements.[12] Arguments are presented concerning the conditions under which these results can be extended to the circular geometry. For the vibration problem it will be assumed that the only effect of the actuator on the plate comes through the applied distributed moments. Mass and stiffness loading of the plate by the actuator is considered negligible. Thus, the results of a static analysis of the strain induced in the plate by the piezoelectric patch can be approximately applied in the dynamic model as oscillating loads.

In rectangular coordinates it was shown that the strains in the plate induced by the piezoelectric element are equal in the two directions,[12]

$$\epsilon_x = \epsilon_y = \frac{-(1+\nu_{pe})P}{1+\nu_p-(1+\nu_{pe})P}\,\epsilon_{pe} \tag{1}$$

where the subscripts pe and p stand for piezoelectric and plate, respectively, $\epsilon_{pe} = d_{31}V/t$ is the unconstrained piezoelectric strain, and

$$P = -\frac{E_{pe}}{E_p}\,\frac{1-\nu_p^2}{\nu_{pe}^2}\,\frac{3ht(2h+t)}{2(h^3+t^3)+3ht^2} \tag{2}$$

Here, the thickness of the plate is $2h$, E is Young's modulus, and ν Poisson's ratio. Also, d_{31} is the voltage-strain piezoelectric constant, and V is the applied voltage. Furthermore, the reaction moments across the actuator in the plate are also equal[12]:

$$m_x = m_y = C_0\epsilon_{pe} \tag{3}$$

where

$$C_0 = -E_p\,\frac{1+\nu_{pe}}{1-\nu_p}\,\frac{P}{1+\nu_p-(1+\nu_{pe})P}\,\frac{2h^2}{3} \tag{4}$$

Thus, the plate reacts to the piezoelectric strain with uniformly distributed bending moments across the extent of the actuator.

For a circular plate the equations of motion can be conveniently written in terms of internal plate moments M_r, M_θ, $M_{r\theta}$ and piezoelectric moments m_r, m_θ (Ref. 14) for dynamic excitation:

$$\frac{\partial^2(M_r-m_r)}{\partial r^2}+\frac{2}{r}\frac{\partial(M_r-m_r)}{\partial r}+\frac{1}{r^2}\frac{\partial^2(M_\theta-m_\theta)}{\partial\theta^2}$$
$$-\frac{1}{r}\frac{\partial(M_\theta-m_\theta)}{\partial r}-\frac{2}{r}\frac{\partial^2 M_{r\theta}}{\partial r\partial\theta}-\frac{2}{r^2}\frac{\partial M_{r\theta}}{\partial\theta}$$
$$+m''w = -p(r,\theta,t) \tag{5}$$

where m'' is the area density of the plate, w the transverse plate displacement, and $p(r,\theta,t)$ the external excitation that results in the undesired vibrations. In connection with Eq. (5) it can be stated that due to the uniform straining of the piezoelectric it is reasonable to assume that the actuator will always attempt to create pure bending conditions in the plate independent of the precise actuator shape if the actuator is small relative to the plate and its shape close to rectangular. Thus, if the reaction moments m_x and m_y are still uniformly distributed under the pie-shaped actuator and are equal in the x and y direction, a simple coordinate transformation results in reaction moments m_r and m_θ in polar coordinates that are also uniformly distributed and equal. To show this, we are reminded that a rectangular actuator will induce spherical bending, implying that there will be no twisting moments m_{xy}. Then, since m_x and m_y are equal, the transformation can be written as[14]

$$m_r = m_x\cos^2\theta + m_y\sin^2\theta = m_x = m_y \tag{6}$$

and

$$m_\theta = m_x\sin^2\theta + m_y\cos^2\theta = m_x = m_y \tag{7}$$

The same transformation confirms that the twisting moment $m_{r\theta}$ is zero and hence explains its absence from Eq. (5). Thus, if it is assumed that the preshaped actuator is small in the sense that its shape approaches that of a rectangle, the uniform and equal reaction moments m_r and m_θ approximate the situation reasonably well. The reaction moments per unit length in the plate across the extent of the piezoelectric can be written as

$$m_r = m_\theta = C_0\epsilon_{pe}\big[h(r-R_1)-h(r-R_2)\big]\big[h(\theta-\theta_1)-h(\theta-\theta_1)\big] \tag{8}$$

where the actuator extends between radii R_1 and R_2 and angles θ_1 and θ_2; $h(\ )$ is the unit step function.

When the piezoelectric moments are transferred to the right-hand side of Eq. (5) and the internal moments written in terms of transverse displacements $w(r,\theta,t)$, the equation of motion

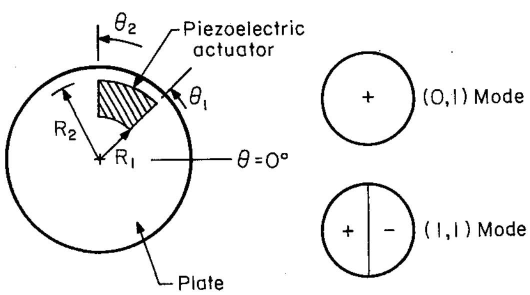

Fig. 1 Piezoelectric actuator configuration (top) and plate lower order modes.

reduces to

$$D\nabla^4 w + m'' w = -p(r,\theta,t) + \frac{\partial^2 m_r}{\partial r^2} + \frac{2}{r}\frac{\partial m_r}{\partial r} + \frac{1}{r^2}\frac{\partial^2 m_\theta}{\partial \theta^2} - \frac{1}{r}\frac{\partial m_\theta}{\partial r} \tag{9}$$

When the piezoelectric-induced terms are written explicitly, after the shown differentiations are performed, it is seen that the effective external loads due to the actuator are

$$\frac{p_{pe}(r,\theta)}{C_0\epsilon_{pe}} = \left[\delta'(r-R_1) - \delta'(r-R_2) + \frac{1}{r}\,\delta(r-R_1) - \frac{1}{r}\,\delta(r-R_2)\right] \times \left[h(\theta-\theta_1) - h(\theta-\theta_2)\right] + \frac{1}{r^2}\left[h(r-R_1)-h(r-R_2)\right]\left[\delta'(\theta-\theta_1)-\delta'(\theta-\theta_2)\right] \tag{10}$$

where $\delta(\,)$ is the unit delta function, and $\delta'(\,)$ indicates its derivative with respect to its argument. Clearly, the actuator then applies 1) uniform line moments along its arc boundaries, 2) uniform transverse forces along these arcs, and 3) line moments along its radial boundaries that vary as $(1/r^2)$.

Let us first consider the piezoelement excitation; the plate response for clamped boundaries can be written in modal terms for harmonic excitation at frequency ω as[15]

$$w(r,\theta) = \sum_{m=0}^{\infty}\sum_{j=1}^{\infty} H_{mj}(k_j^m r)\left[W'_{mj}\cos(m\theta) + W''_{mj}\sin(m\theta)\right] \tag{11}$$

where the radial function is

$$H_{mj}(k_j^m r) = J_m(k_j^m r) - \frac{J_m(k_j^m a)}{I_m(k_j^m a)}\, I_m(k_j^m r) \tag{12}$$

In Eq. (12) the radius of the plate is given by a, and k_j^m are the natural plate wave numbers.[15]

Using standard dynamic analysis techniques, we can show that

$$W'_{mj} = C_0\epsilon_{pe}P'_{mj} \tag{13}$$

where

$$P'_{mj} = \frac{\left(\dfrac{1}{m}\,\phi_{1mj} - m\,\phi_{2mj}\right)\left(\sin m\,\theta_2 - \sin m\,\theta_1\right)}{\Lambda_{mj}\left(\omega_{mj}^2 - \omega^2\right)} \tag{14}$$

and

$$W''_{mj} = C_0\epsilon_{pe}P''_{mj} \tag{15}$$

where

$$P''_{mj} = \frac{\left(\dfrac{1}{m}\,\phi_{1mj} - m\phi_{2mj}\right)\left(\cos m\theta_1 - \cos m\theta_2\right)}{\Lambda_{mj}\left(\omega_{mj}^2 - \omega^2\right)} \tag{16}$$

Also,

$$\phi_{1mj} = (rH'_{mj})\Big|_{r=R_2} - (rH'_{mj})\Big|_{r=R_1} \tag{17}$$

and

$$\phi_{2mj} = \int_{R_1}^{R_2} \frac{H_{mj}}{r}\,\mathrm{d}r \tag{18}$$

For $m=0$ the modal amplitudes are simply

$$W'_{oj} = C_0\epsilon_{pe}\frac{\phi_{1oj}(\theta_2-\theta_1)}{\Lambda_{oj}(\omega_{oj}^2-\omega^2)} \tag{19}$$

$$W''_{oj} = 0 \tag{20}$$

In Eqs. (14) and (16) Λ_{mj} is the normalization constant resulting from the use of the orthogonality property of modes and is given in Ref. 15:

$$\Lambda_{mj} = \epsilon\pi a^2\left[J_m^2(k_j^m a) + J_m'^2(k_j^m a)\right] \tag{21}$$

where $\epsilon=2$ if $m=0$ and $\epsilon=1$ if $m\neq 0$.

We have thus derived an expression for the vibration response of a circular clamped plate as it is excited by the piezoelectric actuators. It is important at this point to stress again the assumptions implicit in this derivation. First, it was assumed that the mass and stiffness loadings of the plate by the bonded elements are negligible. Second, the shape of the actuator closely resembles that of a rectangle and is relatively small relative to the plate size.

Piezoelectric Induced Sound Radiation

It is now possible to derive expressions for the sound field generated by these vibrations. Hansen and Bies[16] calculated the sound radiated by a circular vibrating panel at a distance R in the far field. Their expression can presently be written as

$$p_{\text{rad}}^{pe} = C_R C_0 \epsilon_{pe} A_1 \tag{22}$$

where

$$C_R = \frac{\rho\omega^2 a^2 e^{i(\omega t - kR)}}{R} \tag{23}$$

The expression $k=\omega/c$ is the incident wave number, and

$$A_1 = \sum_{m=0}^{\infty}\sum_{j=1}^{\infty} -\, i^m \Delta_{1mj}(\alpha)\left[P'_{mj}\cos(m\theta') + P''_{mj}\sin(m\theta')\right] \tag{24}$$

Here R, θ' and α are the spherical coordinates shown in Fig. 2, and

$$\Delta_{1mj}(\alpha) = \int_0^1 H_{mj}(k_j^m a\, x) J_m(kax\,\sin\alpha)\, x\,\mathrm{d}x \tag{25}$$

The integral in Eq. (25) has been calculated in closed form in Ref. 16.

Although the control action is applied in the vibration field, one can view the resulting controlled sound field as the superposition of p_{rad}^{pe} and the primary field $p_{\text{rad}}^{\text{inc}}$. In this case the primary field will be created by a plane wave incident from one side of the plate and then partially transmitted as $p_{\text{rad}}^{\text{inc}}$ through plate vibrations. It is these vibrations that we try to alter in a way that will minimize the transmitted sound field power globally.

It was shown by Fuller[5,6] that, when a plane sound wave is incident on a clamped baffled circular plate at an angle θ_i, the

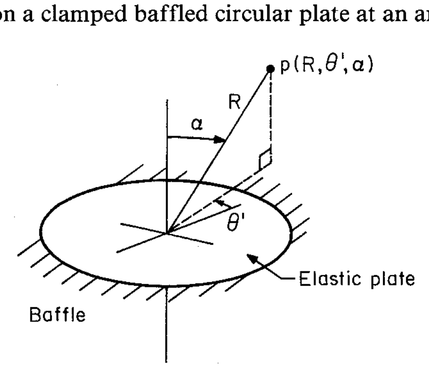

Fig. 2 Radiation coordinate system.

sound pressure transmitted in the far field is given by

$$p_{\rm rad}^{\rm inc} = C_R p_0 B_1 \tag{26}$$

where p_0 is the incident pressure amplitude, and

$$B_1 = 2\pi a^2 \sum_{m=0}^{\infty} \sum_{j=1}^{\infty} \frac{-\epsilon_m \Delta_{1mj}(\alpha)\Delta_{2mj}(\theta_i)}{\Lambda_{mj}(\omega_{mj}^2 - \omega^2)} \cos(m\theta') \tag{27}$$

Here,

$$\Delta_{2mj}(\theta_i) = \int_0^1 H_{mj}(k_j^m a x) J_m(ka x \cos\theta_i) x \, dx \tag{28}$$

is an integral similar to that of Eq. (25), and $\epsilon_m = 1$ if $m = 0$, $\epsilon_m = 2$ if $m \neq 0$.

The sound pressure given in Eq. (26) is to be controlled by the piezoelectric excitation. The task thus reduces to calculating the optimal amplitude and phase of the oscillating control voltage that minimizes the sound power transmitted or radiated from the plate.

Optimal Control Voltage

To determine the optimal complex amplitude of the control voltage, a cost function has to be defined and minimized. For global radiated sound reductions it is best to choose the integral of the squared pressure amplitude over a hemisphere in the far field at radius R. Such a function is proportional to the total radiated power and is quadratic, thus having a single minimum. The cost function can be written as

$$D(C_0\epsilon_{pe}) = \int_S |p_{\rm rad}^{\rm tot}(R,\theta',\alpha)|^2 \, dS \tag{29}$$

The total radiated pressure due to the combined action of the incident sound wave and the piezoelectric action is

$$p_{\rm rad}^{\rm tot} = p_{\rm rad}^{pe} + p_{\rm rad}^{\rm inc} = C_R(C_0\epsilon_{pe}A_1 + p_0 B_1) \tag{30}$$

The cost function is clearly a quadratic function of $(C_0\epsilon_{pe})$ and can be written in an expanded form as

$$\frac{D(C_0\epsilon_{pe})}{\rho^2\omega^4 a^4} = |C_0\epsilon_{pe}|^2 I_A + 2Re\left[C_0\epsilon_{pe} p_0 I_{AB}\right] + P_0^2 I_B \tag{31}$$

where

$$I_A = \int_0^{2\pi}\int_0^{\pi/2} |A_1|^2 \sin\alpha \, d\alpha \, d\theta' \tag{32}$$

$$I_B = \int_0^{2\pi}\int_0^{\pi/2} |B_1|^2 \sin\alpha \, d\alpha \, d\theta' \tag{33}$$

$$I_{AB} = \int_0^{2\pi}\int_0^{\pi/2} A_1 B_1^* \sin\alpha \, d\alpha \, d\theta' \tag{34}$$

Table 1 Material properties

E^p, N/m^2	ρ, kg/m^3	ν	C_L^s, m/s	a, m	$2h$, m
207.10^9	7870	0.29	2916 (shear)	0.2	$1.5875\cdot 10^{-3}$

The minimization procedure for such a cost function was developed by Lester and Fuller.[17] In this case it can be shown that the cost function in Eq. (29) is minimized by (for a single actuator)

$$(C_0\epsilon_{pe})_{\rm opt} = -p_0 \frac{I_{AB}^*}{I_A} \tag{35}$$

When $(C_0\epsilon_{pe})_{\rm opt}$ is substituted into Eq. (31), we get

$$\left(\frac{D(C_0\epsilon_{pe})}{\rho^2\omega^4 a^4}\right)_{\rm opt} = p_0^2\left[I_B - \frac{|I_{AB}|^2}{I_A}\right] \tag{36}$$

It is now possible to begin a parametric investigation concerning the potential of piezoelectric actuators to control sound fields radiated/transmitted by a plate.

Results and Discussion

The physical properties of the plate employed in the example calculations are summarized in Table 1. The acoustic medium was air, and for the following presentation the acoustic plane wave was assumed to be incident to the plate at $\theta_i = 45$ deg. The dimensions and location of the actuator elements will be given for each case studied.

The first step in the calculations was to solve the plate eigenvalue problem for the natural wave number k_j^m, using the Newton-Raphson technique. The first 36 resonant frequencies, ω_{mj}, of the clamped circular thin plate are given in Table 2. All computer simulations presented here will be conducted at the nondimensional frequencies $k_0 a = 0.21$, 0.3265, and 0.45, which are close to the resonant frequency of the fundamental (0,1) mode with $k_{01}a = 0.2175$, in between the first two modes and close to the (1,1) mode with $k_{11}a = 0.45186$, respectively. The infinite series in the expressions for A_1 and B_1 were truncated so that no more than 36 terms were calculated each time; this number was found to be more than adequate for the convergence of all the series involved. The integrals over the spherical coordinate α were performed numerically by a three-point Simpson's rule. Lacking three-dimensional plots, it was decided to present results along the $\theta = 0$ and $\theta = \pi$ radii to facilitate comparisons. Thus, in the displacement plots negative values of the independent variable r/a correspond to the $\theta = \pi$ radius, and the positive ones correspond to the $\theta = 0$ radius. Pressure radiation results are normalized to the constant $|RC_R|$ given by Eq. (23). Vibration plots are normalized to the largest value encountered in the primary and the controlled vibration fields. All calculations were performed on an IBM 3090 mainframe computer.

$R_1 = 0.15$ m, $R_2 = 0.18$ m, $\theta_1 = -15.0$ deg, $\theta_2 = 15.0$ deg

Notice here that the size of the actuator was kept small relative to the plate size ($a = 0.2$ m) and its shape was approximately rectangular. Figure 3 shows the radiated field directivity function in the farfield when the excitation frequency is close to the fundamental. As expected, the uncontrolled field has a monopole character. Clearly, the incident wave excites mainly the (0,1) mode; in addition, that mode has the highest radiation efficiency of all modes.[16] When the optimized voltage to the piezoelectric elements is turned on, the residual

Table 2 Plate resonant frequencies, ω_{mj} (rad/s)

	j					
m	1	2	3	4	5	6
0	372.3669	1449.6584	3247.8462	5765.8149	9003.3852	12,950.5020
1	774.9415	2217.2052	4376.8884	7255.4960	10,853.3567	15,170.5867
2	1271.2680	3083.0377	5606.5601	8847.1647	12,806.1966	17,484.1565
3	1860.0449	4046.7307	6936.5731	10,540.6077	14,861.7412	19,901.0824
4	2538.3197	5106.9334	8365.9521	12,335.0901	17,019.4185	22,420.9070
5	3307.4362	6262.2191	9893.5577	14,229.6978	19,278.4839	25,043.0145

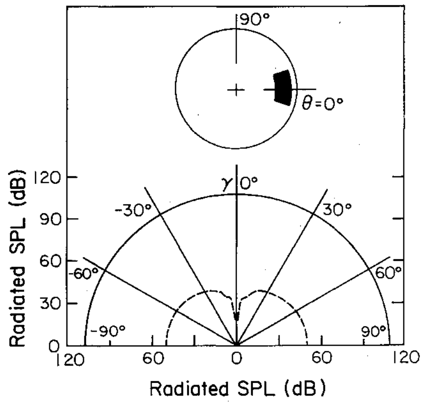

Fig. 3 Radiation directivity: $\omega=360.15$ rad/s, $R_1=0.15$ m, $R_2=0.18$ m, $\theta_1=-15$ deg, $\theta_2=15$ deg (____, uncontrolled; -----, controlled).

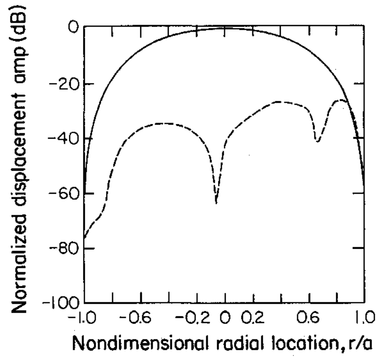

Fig. 4 Displacement distribution: $\omega=360.15$ rad/s, $R_1=0.15$ m, $R_2=0.18$ m, $\theta_1=-15$ deg, $\theta_2=15$ deg (____, uncontrolled; -----, controlled).

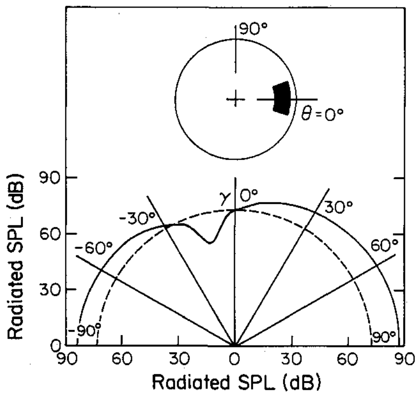

Fig. 5 Radiation directivity: $\omega=771.75$ rad/s, $R_1=0.15$ m, $R_2=0.18$ m, $\theta_1=-15$ deg, $\theta_2=15$ deg (____, uncontrolled; -----, controlled).

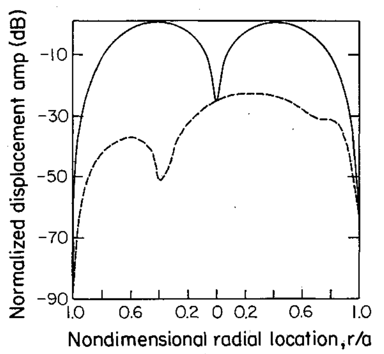

Fig. 6 Displacement distribution: $\omega=771.75$ rad/s, $R_1=0.15$ m, $R_2=0.18$ m, $\theta_1=-15$ deg, $\theta_2=15$ deg (____, uncontrolled; -----, controlled).

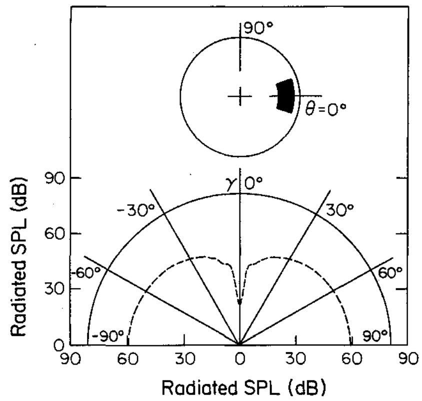

Fig. 7 Radiation directivity: $\omega=560$ rad/s, $R_1=0.15$ m, $R_2=0.18$ m, $\theta_1=-15$ deg, $\theta_2=15$ deg (____, uncontrolled; -----, controlled).

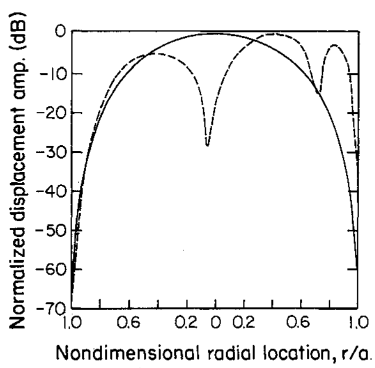

Fig. 8 Displacement distribution: $\omega=560$ rad/s, $R_1=0.15$ m, $R_2=0.18$ m, $\theta_1=-15$ deg, $\theta_2=15$ deg (____, uncontrolled; -----, controlled).

radiation pattern being of a dipole character indicates the suppression of the (0,1) mode; the remaining vibration is mainly in the (1,1) mode. It is seen that the (0,1) mode is essentially completely controlled, with a reduction of at least 55 dB in the sound pressure level.

It is also interesting to examine the plate displacement profile for the uncontrolled and controlled case. Figure 4 shows the predominantly (0,1) modal displacement distribution due to the incident pressure and the remaining vibration after the control has been turned on. The controlled vibration displacement is seen to consist of higher modal components with a (1,1) mode domination; this explains the residual dipole sound field in Fig. 3. It is also interesting to notice the asymmetry in the controlled vibration field; it corresponds to the asymmetry of the control excitation. In connection to that, it is observed that in the residual displacement field the actuator tries to force a node near its boundary at $r/a=0.75$. This result corresponds with the fact that a line moment on a plate will tend to cause rotation of the plate about that line and no displacement.

Figures 5 and 6 show the corresponding sound field directivity patterns and the vibration displacement when the excitation frequency is close to the (1,1) mode natural frequency. In both the primary sound and plate displacement fields, the (1,1) mode dominates. The controlled sound field has a predominately (0,1) monopole character and, as seen in the controlled vibration field, there is a combination of the (0,1) and higher modes. The plate vibration is being reduced by at least 30 dB, whereas the sound field is reduced by about 15 dB over much of the hemisphere. For a portion of the hemisphere, however, "control spillover" into the (0,1) mode actually amplifies radiation; the (0,1) mode has a higher radiation efficiency.[16] Thus, although the (1,1) mode is well controlled, spillover to a strongly radiating (0,1) mode limits the final sound reduction.

Let us switch now to a frequency that is between the first two natural vibration modes. In Fig. 7 the radiation pattern is predominantly of (0,1) character, whereas the controlled field is seen to contain the (1,1) mode only. The sound reduction is at least 20 dB. The vibration results in Fig. 8 explain the situation. On one hand, it seems that the (0,1) mode is well controlled, but there is a great deal of control spillover into the (1,1) mode, which shows in the radiation pattern. Here the actuator is trying to compromise between two conflicting effects. As the actuator tries to reduce the efficient radiator,[16] the (0,1) mode, it spills over strongly into the less efficient (1,1) mode to which it couples well. When a balance is established between the reduction of the (0,1) radiation and the increase of the (1,1) radiation, the control action is optimal. Clearly, there is no significant reduction in the total vibration energy in the plate, but the redistribution of that energy caused by the actuator makes the plate a far less efficient radiator. This result underlines the subtlety of the approach. High sound reduction as been achieved with little change in the spatially averaged plate response. For better results one would need to employ more than one actuator, a need that is seen to be even stronger for point actuators.[5,6] Also notice the node that the actuator is again trying to force near its boundary at $r/a=0.75$. This was also observed in a previous case. The implication is that there is such a general tendency with this type of actuator, especially in off-resonance conditions.

$R_1=0.15$ m, $R_2=0.18$ m, $\theta_1=75$ deg, $\theta_2=105$ deg

Here we have taken the same piezo actuator and placed it at a different location, 90 deg away from its previous position. Radiation results for excitation close to the (0,1) mode are shown in Fig. 9. The reduction here is much stronger than the previous results in Fig. 3; it varies between 75 and 100 dB. An examination of the radiation results over other diagonals reveals that the effect is global. This is not very surprising when one studies the vibration distribution (not shown here). The primary vibration field has a node along the $\theta=90$ deg, 270 deg diam. Therefore, the actuator will couple very poorly to the (1,1) mode. So, while the (0,1) mode is very well controlled, spillover to the (1,1) mode is negligible.

When the excitation frequency is close to the (1,1) frequency, it is again clear in Fig. 10 (the two curves essentially coincide) that the actuator does not couple into that mode and hence it is unable to control it. Apparently, the (1,1) mode created by the actuator is rotated by close to 90 deg with respect to the (1,1) mode forced by the incident wave.

At the intermediate frequency, the (0,1) mode is very well controlled by about 35 dB, as shown in Fig. 11. Again the sound reduction is global over all angles of θ. The reduced spillover into the (1,1) mode results in an overall picture that is better than that for the previous location of the actuator. To see that, compare with the previous results shown in Fig. 7.

$R_1=0.10$, $R_2=0.18$, $\theta_1=-15$ deg, $\theta_2=15$ deg

The actuator here is located at the same position as the first case, but its size is enlarged. It is realized that the chosen size and shape of this actuator will, to a certain degree, violate the assumption of small closely rectangular actuator shape. However, this is clearly acceptable here if we understand that we are only looking for some qualitative insight into the effect of the actuator size. Figure 12 shows a representative result corresponding to the intermediate excitation frequency. It is observed that there is an overall deterioration by about 5 dB over the small actuator. This may indicate that there is an optimal size/location for the actuator. A more extensive theory is being developed in order to attempt an in depth investigation of the size effect.

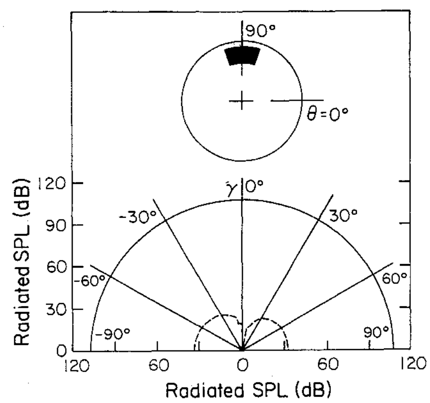

Fig. 9 Radiation directivity: $\omega=360.15$ rad/s, $R_1=0.15$ m, $R_2=0.18$ m, $\theta_1=75$ deg, $\theta_2=105$ deg (____, uncontrolled; -----, controlled).

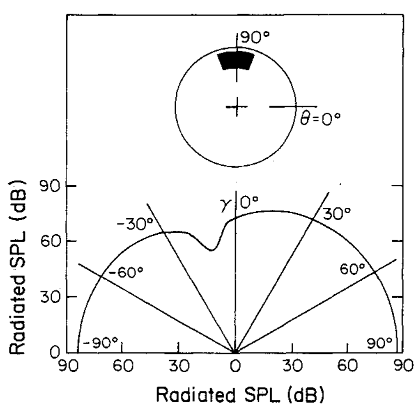

Fig. 10 Radiation directivity: $\omega=771.75$ rad/s, $R_1=0.15$ m, $R_2=0.18$ m, $\theta_1=75$ deg, $\theta_2=105$ deg (____, uncontrolled; -----, controlled).

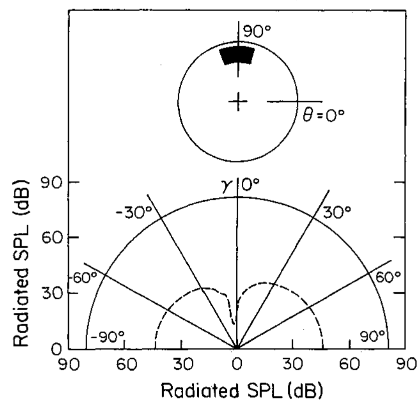

Fig. 11 Radiation directivity: $\omega=560$ rad/s, $R_1=0.15$ m, $R_2=0.18$ m, $\theta_1=75$ deg, $\theta_2=105$ deg (____, uncontrolled; -----, controlled).

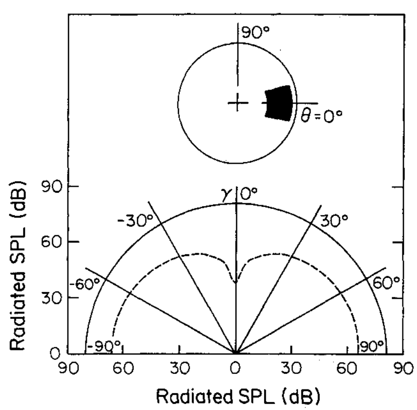

Fig. 12 Radiation directivity: $\omega=560$ rad/s, $R_1=0.15$ m, $R_2=0.18$ m, $\theta_1=-15$ deg, $\theta_2=15$ deg (____, uncontrolled; -----, controlled).

Concluding Discussion

Through theoretical simulations, it has been demonstrated that piezoelectric elements bonded to a plate can be employed to reduce the harmonic sound transmitted or radiated from the plate. The primary field was the sound wave transmitted through the plate when a plane wave is incident from one of its sides. It was shown that global sound attenuation can be achieved near as well as off plate resonances when the actuator complex voltage amplitude is optimized in order to minimize the total sound power radiated to the far field.

During the course of this study, plots were produced that showed the noise fields along the $\theta = \pi/2$ and $\theta = 3\pi/2$ line, that is, normal to the line along which the presented plots were made. These results are not presented here for the sake of brevity, and it suffices to say that they confirmed the global effect of the control in a way very consistent with the presented results.

Through the examples presented, it has become clear that both size and location of the actuator may strongly affect the control potential. This was shown to be the case from both the amount of sound attenuation as well as from the control spillover point of view. Both the degree of wanted coupling to the controlled mode and the lack of coupling to other residual modes are strongly affected by size, shape, and location of the actuator. In addition, considering the tendency of the actuator to force nodes near its boundaries, it may be said that the most effective location of an actuator would be such that its boundaries extend along nodal lines of the vibration field that is to be supressed. For a complete quantitative study of such parameters in order to design the "optimally tailored" actuator, a more accurate analysis is under development in order to overcome the limiting assumptions of the present work.

Other strategies will also have to be addressed with the more accurate models. For example, it would be relatively easy to have more than one actuator at different locations but have them work in phase or at a 180-deg phase difference; such arrangements should improve the coupling into the controlled modes and also reduce spillover in some cases. Finally, control by more than one independently activated actuator will be a natural extension of the present work.

Acknowledgments

The authors are grateful to the Office of Naval Research for their support of this work under grant ONR N00014-88-K-0721. Discussions with C. Rogers of Virginia Polytechnic Institute and State University were also most helpful.

References

[1]Mixson, J. S., and Powell, C. A., "Review of Recent Research on Interior Noise of Propeller Aircraft," AIAA Paper 84-2349, 1984.

[2]Ffowcs-Williams, J. E., "Anti-Sound," *Proceedings of the Royal Society of London, Series A,* Vol. 395, 1984, pp. 63–88.

[3]Warnaka, G. E., "Active Attenuation of Sound," *Noise Control Engineering,* Vol. 18, No. 3, 1982, pp. 100–110.

[4]Mangiante, G. A., "Active Sound Absorption," *Journal of the Acoustical Society of America,* Vol. 61, No. 6, 1977, pp. 1516–1623.

[5]Fuller, C. R., "Active Control of Sound Transmission/Radiation from Elastic Plates by Vibration Inputs, I. Analysis," *Journal of Sound and Vibration,* Vol. 136, No. 1, 1990, pp. 1–15.

[6]Fuller, C. R., "Analysis of Active Control of Sound Radiation from Elastic Plates by Force Inputs," *Proceedings of Inter-Noise 88,* Avignon, France, 1988, pp. 1061–1064.

[7]Fuller, C. R., Silcox, R. J., Brown, D., and Metcalf, V., "Experiments on Structural Control of Sound Transmitted Through an Elastic Plate," *Proceedings of American Control Conference,* Pittsburgh, PA, 1989, pp. 2079–2089.

[8]Crawley, E. F., and de Luis, J., "Use of Piezoelectric Actuators as Elements of Intelligent Structures," *AIAA Journal,* Vol. 25, No. 10, 1987, pp. 1373–1385.

[9]Bailey, T., and Hubbard, J. E., "Distributed Piezoelectric-Polymer Active Vibration Control of a Cylinder Beam," *Journal of Guidance, Control, and Dynamics,* Vol. 6, No. 5, 1985, pp. 605–611.

[10]Fanson, J. L., and Chen, J. C., "Structural Control by the Use of Piezoelectric Active Members," *Proceedings of NASA/DOD Control-Structures Interaction Conference,* NASA CP-2447, Pt. II, 1986.

[11]Baz, A., and Poh, S., "Optimum Vibration Control of Flexible Beams by Piezoelectric Actuators," NASA CR-180209, 1987.

[12]Dimitriadis, E. K., Fuller, C. R., and Rogers, C. A., "Piezoelectric Actuators for Distributed Vibration Excitation of Thin Plates," *Journal of Vibration and Acoustics,* Vol. 113, 1991, pp. 100–107.

[13]Fuller, C. R., Hansen, C. H., and Snyder, S. D., "Active Control of Structurally Radiated Noise Using Piezoceramic Actuators." *Proceedings of Inter-Noise '89,* Long Beach, CA, 1989, pp. 509–512.

[14]Timoshenko, S. P., and Woinowsky-Krieger, J., *Theory of Plates and Shells,* McGraw-Hill, New York, 1959.

[15]Morse, P. M., and Ingard, K. U., *Theoretical Acoustics,* Princeton Univ. Press, Princeton, NJ, 1986.

[16]Hansen, C. H., and Bies, D. A., "Optical Holography for the Study of Sound Radiation from Virbrating Surfaces," *Journal of the Acoustical Society of America,* Vol. 60, No. 3, 1976, pp. 543–555.

[17]Lester, H. C., and Fuller, C. R., "Active Control of Propeller Induced Noise Fields Inside a Flexible Cylinder," AIAA Paper 86-1957, 1986.